Bulk Gold = Yellow

Nanogold = Red
Quantum Confinement

3D confinement (Quantum dots)
- Fullerenes
- Colloidal particles
- Nanoporous silicon
- Activated carbons
- Nitride and carbide precipitates in high-strength low-alloy steels
- Semiconductor particles in a glass matrix for non-linear optical components
- Semiconductor quantum dots (self-assembled and colloidal)

2D confinement (Nanowires)
- Carbon nanotubes and nanofilaments
- Metal and magnetic nanowires
- Oxide and carbide nanorods
- Semiconductor quantum wires

1D confinement (Quantum layer or well)
- Nanolaminated or compositionally modulated materials
- Grain boundary films
- Clay platelets
- Semiconductor quantum wells and superlattices
- Magnetic multilayers
- Langmuir-Blodgett films
- Silicon inversion layers in field effect transistors
- Surface-engineered materials for increased wear resistance or corrosion resistance
Quantum Confinement in Nanostructures

Confined in:

1 Direction: Quantum well (thin film)

Two-dimensional electrons

2 Directions: Quantum wire

One-dimensional electrons

3 Directions: Quantum dot

Zero-dimensional electrons

Each confinement direction converts a continuous $k$ into a discrete quantum number $n$. 
CdSe nanocrystal size and band gap

Photon Energy and Wavelength

$$h\nu (eV) = \frac{1240}{\lambda (nm)}$$

Quantum dot
Zero-dimensional electrons
Quantum confinement in 3D
Zero-D material

CdSe Quantum Dots (Bawendi Group)

Photo by Felice Frankel
Quantum Wire

One-dimensional electrons

Quantum confinement in 2D

1D material

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CdS Quantum Layers

![Graph showing absorbance over wavelength and 2θ degrees with time progression for CdS Quantum Layers.](image-url)
Bi$_2$Te$_3$ Nanowires, nanobelts and nanofilms
Time-dependant STM and AFM images and cross-sectional profiles of Bi 2 Te 3 nanofilms formed at +50 mV.

For each figure, the lower panel represents the cross-sectional profiles along the arrow.

pH: 1.5
Without EDTA
RA-FTIR spectra of Bi₂Te₃ nanostructures deposited in acidic medium at different deposition times

Band gap dependency of Bi₂Te₃ nanofilms as a function of the thickness

The XRD patterns of the Bi₂Te₃ nanofilms electrodeposited onto single crystal Au (1 1 1) in pH: 1.5 at the different deposition times: (a) 15 and (b) 35 min

Rhombohedral Bi₂Te₃ with hexagonal structure, preferred growth direction (015)
RA-FTIR spectra of Bi$_2$Te$_3$ nanostructures deposited in basic medium at different deposition times

The XRD patterns of the Bi$_2$Te$_3$ nanowires electrodeposited onto single crystal Au (1 1 1) in pH: 9.0 at the different deposition times: (a) 30 and (b) 55 min.

Band gap dependency of Bi$_2$Te$_3$ nanostructures as a function of the thickness.
\( E = \frac{\hbar^2}{8r^2} \left( \frac{1}{m^*_c} + \frac{1}{m^*_h} \right) \)

\( E_n = \frac{n^2 \hbar^2}{8m} \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) \)

\( \Delta E(r) = E_{\text{gap}} + \left( \frac{\hbar^2}{8r^2} \right) \left( \frac{1}{m^*_c} + \frac{1}{m^*_h} \right) \)

**Bi\(_2\)Te\(_3\) Nanowires, nanobelts and nanofilms**

**Bi\(_2\)Te\(_3\)**

Electrodeposition

pH: 9 with EDTA

pH: 1.5 without EDTA

0.8

0.7

0.6

0.5

0.4

0.3

0.2

0.1

0

20

40

60

80

100

120

**Band gap (eV)**

**Thickness (nm)**

- acidic
- basic

Nanowire

Nanofilm

Nanobelt

1 D Confinement

2 D Confinement
The properties of a material depend on the type of motion its electrons can execute, which depends on the space available for them. Thus, the properties of a material are characterized by a specific “length scale”, usually on the nm dimension. If the physical size of the material is reduced below this length scale, its properties change and become sensitive to size and shape.

Size effects constitute a peculiar and fascinating aspect of nanomaterials. The effects determined by size pertain to the evolution of structural, thermodynamic, electronic, spectroscopic, electromagnetic and chemical features of these finite systems with changing size.
Democritus, 400 BC

He asked: Could matter be divided into smaller and smaller pieces forever, or was there a limit to the number of times a piece of matter could be divided?

- His theory: Matter could not be divided into smaller and smaller pieces forever, eventually the smallest possible piece would be obtained.
- This piece would be indivisible.
- He named the smallest piece of matter “atomos,” meaning “not to be cut.”

To Democritus, atoms were small, hard particles that were all made of the same material but were different shapes and sizes.

This theory was ignored and forgotten for more than 2000 years!
Democritus’s ideas were rejected later by Aristotle who believed that matter could be endlessly divided. Aristotle supported the “earth, air, water, and fire” concept of matter.

384-322BC
Aristotle and fellow Greeks

Aristotle and Plato favored the earth, fire, air and water approach to the nature of matter. Their ideas held sway because of their eminence as philosophers. The atomos idea was buried for approximately 2000 years.

The eminent philosophers of the time, Aristotle and Plato, had a more respected, (and ultimately wrong) theory.
Alchemy is a mixture of science and mysticism. Lab procedures were developed, but alchemists did not perform controlled experiments like true scientists.

Alchemists believed that cheap metals could be turned into gold (Philosopher’s Stone). Alchemists were almost like magicians (elixirs, physical immortality).

They wanted to live forever, so they started out with trying to make a potion that could make them live forever. They didn’t succeed, but they did make many experiments, and the scientific method.

They also kept careful records.

Alchemy is a mixture of science and mysticism. Lab procedures were developed, but alchemists did not perform controlled experiments like true scientists.

Modern chemistry evolved from alchemy.
Dalton’s Atomic Theory (1808)

Building on the ideas of Democritus in ancient Greece.

- He deduced that all **elements** are composed of atoms. Atoms are indivisible and indestructible particles.
- Atoms of the **same** element are exactly alike.
- Atoms of **different** elements are **different**.
- **Compounds** are formed by the joining of atoms of two or more elements.
- Atoms of different elements combine together in **simple proportions** to create compounds.
- In a chemical reaction, atoms are rearranged, but not changed.

This theory became one of the **foundations of modern chemistry**.
Thomson Model

Discovery of Electrons

Thomson studied the **passage** of an electric current through a gas. As the current passed through the gas, it gave off rays of **negatively charged particles.**

Thomson concluded that the negative charges came from **within** the atom. Thomson called the negatively charged “**corpuscles,**” today known as electrons.

A particle smaller than an atom **had to exist.**

The atom was **divisible!**
He proposed a model of the atom that is sometimes called the “Plum Pudding” model.

Atoms were made from a positively charged substance with negatively charged electrons scattered about, like raisins in a pudding.

Plum-pudding Model
- positive sphere (pudding) with negative electrons (plums) dispersed throughout

Since the gas was known to be neutral, having no charge, he reasoned that there must be positively charged particles in the atom. But he could never find them.

J.J. Thomson, measured mass/charge of $e^-$

$$\frac{e}{m} = 1.75888 \times 10^{11} \text{ C/Kg}$$

(1906 Nobel Prize in Physics)
Oil sprayed in fine droplets

Pinhole

X rays to produce charge on oil droplet

Electrically charged brass plates

Telescopic eyepiece

Charged oil droplet under observation

1.593 \times 10^{-19} \text{ C} 

1.602 \times 10^{-19} \text{ C}

9.109 \times 10^{-31} \text{ kg}
Rutherford’s experiment (1911).

He knew that atoms had positive and negative particles, but could not decide how they were arranged.

Rutherford’s experiment involved firing a stream of tiny positively charged particles at a thin sheet of gold foil (2000 atoms thick) to isolate the positive particles in an atom.

Results showed that the atoms were mostly empty space, but had a dense central core. (gold foil exp.)
This could only mean that the gold atoms in the sheet were mostly open space. Atoms were not a pudding filled with a positively charged material.

Rutherford concluded that an atom had a small, dense, positively charged center that repelled his positively charged "bullets."

He called the center of the atom the "nucleus"

The nucleus is tiny compared to the atom as a whole.

- Rutherford – Solar System
  – Why? Scattering showed a small, hard core.
  Problem: electrons should spiral into nucleus in \( \sim 10^{-11} \) sec.
Bohr was trying to show why the negative electrons were not sucked into the nucleus of the atom.

Proposed that electrons traveled in fixed paths around the nucleus. Scientists still use the Bohr model to show the number of electrons in each orbit around the nucleus.

The size of the allowed electron orbits is determined by a condition imposed on the electron’s orbital angular momentum: the allowed orbits are those for which the electron’s orbital angular momentum about the nucleus is an integral multiple of $\hbar$ (pronounced “$h$ bar”), where $\hbar = h/2\pi$:

$$m_e vr = n\hbar \quad n = 1, 2, 3, \ldots$$ [28.4]
The electrical potential energy of the atom is

\[ PE = k_e \frac{q_1 q_2}{r} = k_e \frac{(-e)(e)}{r} = -k_e \frac{e^2}{r} \]

where \( k_e \) is the Coulomb constant. Assuming the nucleus is at rest, the total energy \( E \) of the atom is the sum of the kinetic and potential energy:

\[ E = KE + PE = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r} \]

[28.5]

We apply Newton’s second law to the electron. We know that the electric force of attraction on the electron, \( k_e e^2 / r^2 \), must equal \( m_e a_r \), where \( a_r = v^2 / r \) is the centripetal acceleration of the electron. Thus,

\[ k_e \frac{e^2}{r^2} = m_e \frac{v^2}{r} \]

[28.6]
From this equation, we see that the kinetic energy of the electron is

\[ \frac{1}{2} m_e v^2 = \frac{k_e e^2}{2r} \]  \[28.7\]

We can combine this result with Equation 28.5 and express the energy of the atom as

\[ E = -\frac{k_e e^2}{2r} \]  \[28.8\]

where the negative value of the energy indicates that the electron is bound to the proton.

An expression for \( r \) is obtained by solving Equations 28.4 and 28.6 for \( v \) and equating the results:

\[ m_e v r = n \hbar \quad n = 1, 2, 3, \ldots \]

\[ v^2 = \frac{n^2 \hbar}{m_e r^2} = \frac{k_e e^2}{m_e r} \]

\[ k_e \frac{e^2}{r^2} = m_e \frac{v^2}{r} \]

\[ r_n = \frac{n^2 \hbar^2}{m_e k_e e^2} \quad n = 1, 2, 3, \ldots \] \[28.9\]

This equation is based on the assumption that the electron can exist only in certain allowed orbits determined by the integer \( n \).
The orbit with the smallest radius, called the Bohr radius, \( a_0 \), corresponds to \( n = 1 \) and has the value

\[
a_0 = \frac{\hbar^2}{mk_e e^2} = 0.052 \text{ nm} \tag{28.10}
\]

A general expression for the radius of any orbit in the hydrogen atom is obtained by substituting Equation 28.10 into Equation 28.9:

\[
r_n = n^2 a_0 = n^2 (0.052 \text{ nm}) \tag{28.11}
\]

The first three Bohr orbits for hydrogen are shown in Active Figure 28.6.

Equation 28.9 may be substituted into Equation 28.8 to give the following expression for the energies of the quantum states:

\[
E_n = -\frac{m_e k_e^2 e^4}{2\hbar^2} \left( \frac{1}{n^2} \right) \quad n = 1, 2, 3, \ldots \tag{28.12}
\]

If we insert numerical values into Equation 28.12, we obtain

\[
E_n = -\frac{13.6}{n^2} \text{ eV} \tag{28.13}
\]

The analysis used in the Bohr theory is also successful when applied to hydrogen-like atoms. An atom is said to be hydrogen-like when it contains only one electron. Examples are singly ionized helium, doubly ionized lithium, triply ionized beryllium, and so forth. The results of the Bohr theory for hydrogen can be extended to hydrogen-like atoms by substituting \( Ze^2 \) for \( e^2 \) in the hydrogen equations, where \( Z \) is the atomic number of the element. For example, Equations 28.12 and 28.15 become

\[
E_n = -\frac{m_e k_e^2 Z^2 e^4}{2\hbar^2} \left( \frac{1}{n^2} \right) \quad n = 1, 2, 3, \ldots \tag{28.18}
\]
His model of the hydrogen atom consists of electrons circling the nucleus but are restricted to particular orbits, like the planets around the Sun.

The electron is in its lowest energy state when it is in the orbit closest to the nucleus. The energy is higher when its in orbits successively farther from the nucleus.

It can move to a higher energy orbit by gaining an amount of energy equal to the difference between the higher energy orbit and the initial energy orbit.

When an electron drops back to a lower energy orbit a photon is emitted that has energy equal to the energy difference between the two orbits.

Bohr’s most important contribution was the concept that the energy of these orbits is quantized.

The shortcoming of Bohr’s model was that it could only explain the atomic spectrum of hydrogen, but could not explain the atomic spectrum of other elements.

Bohr – fixed energy levels
– Problem: No reason for fixed energy levels
Louis de Broglie

For more than a decade following Bohr’s publication, no one was able to explain why the angular momentum of the electron was restricted to these discrete values. He assumed that an electron orbit would be stable (allowed) only if it contained an integral number of electron wavelengths.

- Light sometimes behaves like a wave and sometimes like a particle, then particles could have a wave behavior.
- Louis de Broglie came up with the idea that particles might also have wave properties
  - deBroglie – electron standing waves +
    - Why? Explains fixed energy levels
    - Problem: still only works for Hydrogen
Electrons are not in specific energy levels, but rather in sublevels and orbitals. Created a mathematical model that described electrons as waves.

An orbital is the average region where an electron is most likely to be found. 95% of the time, they can be found in Bohr’s proposed orbits.

Forms an electron cloud.

**Quantum mechanics**
- electrons can only exist in specified energy states

**Electron cloud model**
- *orbital*: region around the nucleus where $e^-$ are likely to be found

Bohr's model was only applicable to hydrogen-like atoms. In 1925, more general forms of description (now called quantum mechanics) emerged, thanks to Heisenberg and Schrodinger.
1932—James Chadwick

“How do all the positive protons stay together when they should repel each other?”

Rutherford predicted that a neutral particle would be found in the nucleus.

- Discovered neutrons to have 0 charge with 1 amu.
- Neutrons are located in the nucleus.
Corpuscular Theory of Light (1704)

- Isaac Newton proposed that light consists of a stream of small particles, because it
  - travels in straight lines at great speeds
  - is reflected from mirrors in a predictable way

Wave Theory of Light (1802)

- Thomas Young showed that light is a wave, because it
  - undergoes diffraction and interference
    (Young’s double-slit experiment)
Particles

Position $x$
Mass $m$
Momentum $p = mv$

A particle is localised in space, and has discrete physical properties such as mass.

Waves

Wavelength $\lambda$
Amplitude $A$
Frequency $f$

- number of cycles per second (Hertz)

$$f = \frac{c}{\lambda}$$  
$$y = A \sin (2\pi ft + \theta)$$

- A wave is inherently spread out over many wave-lengths in space, and could have amplitudes in a continuous range
- Waves superpose and pass through each other, while particles collide and bounce off each other
Diffraction

Interference

Diffraction: spreading out of plane waves as they pass through hole

Constructive Interference occurs where wave crests meet

Destructive Interference occurs where wave crest and trough meet

Interference Fringes on a Screen

Slits

P Bright area

Q Bright area

P

R Dark area

Screen
\[ \delta = d \sin \theta_{\text{bright}} = m\lambda \quad m = 0, \pm 1, \pm 2, \ldots \]

\[ \delta = d \sin \theta_{\text{dark}} = (m + \frac{1}{2})\lambda \quad m = 0, \pm 1, \pm 2, \ldots \]
Blackbody Radiation

• A blackbody is an object which totally absorbs all radiation that falls on it
• Any hot body (blackbodies included) radiates light over the whole spectrum of frequencies
• The spectrum depends on both frequency and temperature

Spectrum of Blackbody Radiation

Ultraviolet Catastrophe

Plot of intensity of the blackbody radiation versus wavelength for various temperatures

Classical theory predicts a graph that deviates from experimental data, especially at short wavelengths
Planck’s Quantum Postulate (1900)

• A blackbody can only emit radiation in discrete packets or quanta, i.e., in multiples of the minimum energy:

\[ E = hf \]

where \( h \) is a constant and \( f \) is the frequency of the radiation

Planck’s Constant

• Experimentally determined to be

\[ h = 6.63 \times 10^{-34} \text{ Joule sec}, \text{ (Joule} = \text{ kg m}^2 / \text{ sec}^2) \]

• A new constant of nature, which turns out to be of fundamental importance in the new ‘quantum theory’
The photoelectric effect and the particle theory of light

Several features of the photoelectric effect can’t be explained with classical physics or with the wave theory of light:

- No electrons are emitted if the incident light frequency falls below some **cutoff frequency** $f_c$, which is characteristic of the material being illuminated. This is inconsistent with the wave theory, which predicts that the photoelectric effect should occur at any frequency, provided the light intensity is sufficiently high.
- The maximum kinetic energy of the photoelectrons is independent of light intensity. According to wave theory, light of higher intensity should carry more energy into the metal per unit time and therefore eject photoelectrons having higher kinetic energies.
- The maximum kinetic energy of the photoelectrons increases with increasing light frequency. The wave theory predicts no relationship between photoelectron energy and incident light frequency.
- Electrons are emitted from the surface almost instantaneously (less than $10^{-9}$ s after the surface is illuminated), even at low light intensities. Classically, we expect the photoelectrons to require some time to absorb the incident radiation before they acquire enough kinetic energy to escape from the metal.
With the photon theory of light, we can explain the previously mentioned features of the photoelectric effect that cannot be understood using concepts of classical physics:

- Photoelectrons are created by absorption of a single photon, so the energy of that photon must be greater than or equal to the work function, else no photoelectrons will be produced. This explains the cutoff frequency.
- From Equation of $KE_{\text{max}} = h\nu - \phi$, $KE_{\text{max}}$ depends only on the frequency of the light and the value of the work function. Light intensity is immaterial, because absorption of a single photon is responsible for the electron’s change in kinetic energy.
- $KE$ is linear in the frequency, so $KE_{\text{max}}$ increases with increasing frequency.
- Electrons are emitted almost instantaneously, regardless of intensity, because the light energy is concentrated in packets rather than spread out in waves. If the frequency is high enough, no time is needed for the electron to gradually acquire sufficient energy to escape the metal.
Compton directed an x-ray beam of wavelength $\lambda_0$ toward a block of graphite. He found that the scattered x-rays had a slightly longer wavelength than the incident x-rays, and hence the energies of the scattered rays were lower. The amount of energy reduction depended on the angle at which the x-rays were scattered. The change in wavelength between a scattered x-ray and an incident x-ray is called the Compton shift.

In order to explain this effect, Compton assumed that if a photon behaves like a particle, its collision with other particles is similar to a collision between two billiard balls. Hence, the x ray photon carries both measurable energy and momentum, and these two quantities must be conserved in a collision. If the incident photon collides with an electron initially at rest, as in Figure 27.16, the photon transfers some of its energy and momentum to the electron. As a consequence, the energy and frequency of the scattered photon are lowered and its wavelength increases.

The Compton wavelength is very small relative to the wavelengths of visible light, so the shift in wavelength would be difficult to detect if visible light were used.
X-RAYS

Continuous radiation (bremsstrahlung)

Nucleus of target atom

Deflected lower energy electron

Incoming electron

Emitted photon

energy difference:

\[ e\Delta V = h_f = \frac{hc}{\lambda_{\text{min}}} \]

\[ \lambda_{\text{min}} = \frac{hc}{e\Delta V} \]

characteristic x-rays
2d \sin \theta = m\lambda \quad (m = 1, 2, 3, \ldots)
THE DUAL NATURE OF LIGHT AND MATTER

In 1924 Einstein wrote:- “There are therefore now two theories of light, both indispensable, and … without any logical connection.”

The Wave Properties
Difraction, Interference etc.

The Particles Properties
• Photoelectric effect
• Compton Scattering

Light has a dual nature, exhibiting both wave and particle characteristics.
The De Broglie Relation

For particles, and particularly for photons, the energy can be given by Einstein’s famous formula:

\[ E = mc^2 = mce = pc \]

But, the energy of the photon is also given by Planck’s famous law:

\[ E = h\nu \]

so that

\[ E = h\nu = h\frac{c}{\lambda} = pc \quad \Rightarrow \quad p = \frac{h}{\lambda} \]

The momentum of the photon is said to be \( mc=p \)

de Broglie’s relation
De Broglie Relation

From the de Broglie relation, we now can find the wavelength for the particle:

\[ \lambda = \frac{h}{p} \]

De Broglie postulated this relation in his thesis, and suggested that quantization in atoms followed by requiring an integer number of wavelengths in the electron orbit. While logical, this view was emphatically challenged by the Copenhagen group.

We must fit an integer number of wavelengths into the orbit.
There still remain differences between particles and waves:

**Particles**

\[ E = \frac{p^2}{2m} \]

\[ \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \]

\[ k = \frac{2\pi}{\lambda} = \frac{\sqrt{2mE}}{\hbar} \]

**Waves**

\[ E = hf \]

\[ \lambda = \frac{c}{f} = \frac{hc}{E} \]

\[ k = \frac{2\pi}{\lambda} = \frac{E}{\hbar c} \]

---

While we use similar notation for particles and for true waves, various quantities are defined differently. Do not make the mistake of using optical definitions for particles.
Let us consider a photon and an electron, both of which have an energy of $1 \text{ eV} \ (1.6 \times 10^{-19} \text{ J})$

\[
\lambda_{\text{photon}} = \frac{c}{f} = \frac{hc}{E} = \frac{(6.62618 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^8 \text{ m/s})}{1.6 \times 10^{-19} \text{ J}} = 1.24 \times 10^{-6} \text{ m}
\]

\[
\lambda_{\text{particle}} = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.62618 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(9.1 \times 10^{-31} \text{ kg})(1.6 \times 10^{-19} \text{ J})}} = 1.23 \times 10^{-9} \text{ m}
\]

There are 3 orders of magnitude difference in the size of the two wavelengths.

The fact that the wavelengths for particles are so small is why they are usually never observed. On the other hand, they become important when we deal with nano-technology, since we are on the same dimensional scale!
Goal  Apply the de Broglie hypothesis to a quantum and a classical object.

Problem  (a) Compare the de Broglie wavelength for an electron \( m_e = 9.11 \times 10^{-31} \text{ kg} \) moving at a speed of \( 1.00 \times 10^7 \text{ m/s} \) with that of a baseball of mass 0.145 kg pitched at 45.0 m/s. (b) Compare these wavelengths with that of an electron traveling at 0.999c.

Strategy  This is a matter of substitution into Equation 27.14 for the de Broglie wavelength. In part (b), the relativistic momentum must be used.

Solution
(a) Compare the de Broglie wavelengths of the electron and the baseball.

Substitute data for the electron into Equation 27.14:

\[
\lambda_e = \frac{\hbar}{m_e v} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^7 \text{ m/s})} = 7.28 \times 10^{-11} \text{ m}
\]

Repeat the calculation with the baseball data:

\[
\lambda_b = \frac{\hbar}{m_b v} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.145 \text{ kg})(45.0 \text{ m/s})} = 1.02 \times 10^{-34} \text{ m}
\]

(b) Find the wavelength for an electron traveling at 0.999c.

Replace the momentum in Equation 27.14 with the relativistic momentum:

\[
\lambda_e = \frac{\hbar}{m_e \sqrt{1 - v^2/c^2}} = \frac{\hbar \sqrt{1 - v^2/c^2}}{m_e v}
\]

Substitute:

\[
\lambda_e = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \sqrt{1 - (0.999c)^2/c^2}}{(9.11 \times 10^{-31} \text{ kg})(0.999\cdot 3.00 \times 10^8 \text{ m/s})} = 1.09 \times 10^{-13} \text{ m}
\]

Remarks  The electron wavelength corresponds to that of x-rays in the electromagnetic spectrum. The baseball, by contrast, has a wavelength much smaller than any aperture through which the baseball could possibly pass, so we couldn’t observe any of its diffraction effects. It is generally true that the wave properties of large-scale objects can’t be observed. Notice that even at extreme relativistic speeds, the electron wavelength is still far larger than the baseball’s.

Exercise 27.7
Find the de Broglie wavelength of a proton \( m_p = 1.67 \times 10^{-27} \text{ kg} \) moving with a speed of \( 1.00 \times 10^7 \text{ m/s} \).

Answer  \( 3.97 \times 10^{-14} \text{ m} \)
Double-Slit Experiment
to illustrate wave nature of light
If electrons were purely PARTICLES what sort of behavior might we expect?

* Think of the case where we fire BULLETS from a gun

* A very distinctive distribution of counts will be obtained

MONITORING THE POSITION AND COUNTING THE NUMBER OF BULLETS THAT STRIKE THE SCREEN WE WOULD EXPECT THE PARTICLE-LIKE DISTRIBUTION SHOWN ABOVE
Double-Slit Experiment

with a machine gun!
Double-Slit Experiment

with electron gun

Electrons behave like waves!

The electrons show an INTERFERENCE PATTERN that is Similar to that found when we perform the same experiment with LIGHT.
The gun is slowed down to emit a *single photon*. Do you expect interference to happen now?

- After many electrons, resembles the interference pattern of light

Interference occurs even with a *single* photon

*Results are the same as for photons*—electrons are a wave!
Double-Slit Experiment with electron gun and detector

With an aim to test from which slit the photon is actually passing to reach the other side, we place a monitor near one of the slits and redo the experiment with single photon.

What do you expect now? Any guesses?

Trying to detect which slit the electrons pass through causes them to behave like particles and the interference pattern disappears

The photon now behaves as a particle! The wave function of the photon has collapsed!!!
Interpretation of Double-Slit Experiment

Interference occurs even with a *single* photon or electron

...can a photon interfere with itself?

The quantum mechanical explanation is that each particle passes through both slits at once and interferes *with itself*.

If we try to find out which slit the particle goes through the interference pattern vanishes!

We cannot see the wave and particle nature at the same time.

If we know which path the particle takes, we lose the fringes.

Which property we “measure” depends upon the type of measurement that is carried out.

The importance of the two-slit experiment has been memorably summarized by Richard Feynman: “...a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the *only* mystery.”

Anyone who is not shocked by quantum mechanics has not understood it. -Niels Bohr
Summary

We are led to conclude that ALL “things”, whether electrons or photons or automobiles or …… are both particle and wave!

Which property we “measure” depends upon the type of measurement that is carried out.

* The two-slit experiment is a wave-like measurement
* The photo-electric effect is a particle-like measurement

This *wave-particle duality* is a part of the principle of complementarity. The complementarity principle is considered as Neils Bohr’s important contribution, yet he had trouble fully accepting a wave-based theory that was due to Schrödinger and the electron wave concept of de Broglie.
Quantum particles are usually delocalized, meaning they do not have a well-specified position.

Classical particle
Position = x

The particle is here.

Quantum particle
Wavefunction = $\psi(x)$

With some high probability, the particle is probably somewhere around here.
wave-particle duality
Heisenberg Uncertainty Principle

- In the world of very small particles, one cannot measure any property of a particle without interacting with it in some way.
- Observations generally require energy interacting with matter (In many cases this is scattering of EM waves).
  - Light on a ruler
  - Radar on a car
  - Touch on a surface
  - Voltmeter in a circuit
- This introduces an unavoidable uncertainty into the result.
- One can never measure all the properties exactly.
Measuring Position and Momentum of an Electron

- Shine light on electron and detect reflected light using a microscope
- Minimum uncertainty in position is given by the wavelength of the light
- So to determine the position accurately, it is necessary to use light with a short wavelength

\[ E = \frac{hc}{\lambda} \]

- By Planck’s law \( E = hc/\lambda \), a photon with a short wavelength has a large energy
- Thus, it would impart a large ‘kick’ to the electron
- But to determine its momentum accurately, electron must only be given a small kick
- This means using light of long wavelength!
Implications

- It is impossible to know both the position and momentum exactly, i.e., $\Delta x=0$ and $\Delta p=0$
- These uncertainties are inherent in the physical world and have nothing to do with the skill of the observer
- Because $\hbar$ is so small, these uncertainties are not observable in normal everyday situations

$$\hbar = 1.054 \times 10^{-34} \text{ [J} \cdot \text{s]}$$

If a measurement of position of made with precision $\Delta x$ and a simultaneously measurement of momentum is made with precision $\Delta p$, then the product of the uncertainties can not be smaller than the order of $\hbar$.

$$\Delta x \Delta p \geq \hbar \text{ (The uncertainty principle)}$$

$$\Delta E \Delta t \geq \hbar$$